

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2018/2019

TMT 1171 – MATHEMATICAL TECHNIQUES 1

(All sections / Groups)

31 MAY 2019
9.00 a.m – 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **4 pages** only with **4 Questions**.
2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write your answers in the Answer Booklet provided, and **start each solution of a question on a new page**.
4. Show all steps.

Question 1 (10 marks)

a) Consider the vector $u = \langle 1, -1, 3 \rangle$ and vector $v = \langle -2, 3, 1 \rangle$,

i) Find the cross product of u and v .

[1.5 marks]

ii) Find all unit vectors that are perpendicular to both u and v .

[1.5 marks]

b) Find the **eigenvalues** for the following **matrix A**:

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

[2 marks]

c) Consider the following linear system:

$$2x_1 + x_2 - x_3 = 0$$

$$x_1 + x_3 = 4$$

$$x_1 + x_2 + x_3 = 0$$

i) Write down an **augmented matrix** for the given linear system.

[0.5 mark]

ii) Turn the **augmented matrix** into **row echelon form** by using **Gaussian Elimination**.

[3 marks]

iii) Use back-substitution to find the values of x_1 , x_2 and x_3 .

[1.5 marks]

Continued.....

Question 2 (15 marks)

a) Evaluate the following limits. Show the necessary steps.

i) $\lim_{x \rightarrow 5} \frac{-2x + 10}{x^2 - 8x + 15}$

[2 marks]

ii) $\lim_{x \rightarrow \infty} \frac{x^3 - 4}{5x^3 + x^2}$

[2 marks]

b) Let $f(x) = \begin{cases} 2x + 3, & x < 2 \\ x^2 + 2x - 1 & x > 2 \\ 3 & x = 2 \end{cases}$

i) Determine $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$.

[1 mark]

ii) Determine $\lim_{x \rightarrow 2} f(x)$.

[1 mark]

iii) Is the function f continuous at 2? Give the reason for your answer.

[1 mark]

c) Find the derivative of the following functions.

i) $y = e^{2x} \sin(3x)$

[1.5 marks]

ii) $y = \frac{e^{2x}}{1+x}$

[1.5 marks]

d) Use implicit differentiation to find $\frac{dy}{dx}$ of the equation $x^2 + 2xy - y^2 + x = 2$.

[2 marks]

e) Given $f(x) = 12x - x^3$, find the absolute maximum value of f over the interval $[0, 3]$.

[3 marks]

Continued.....

Question 3 (10 marks)

a) Compute the integral

$$\int_1^2 \frac{6-3x^2}{\sqrt{6x-x^3}} dx.$$

[3.5 marks]

b) Use integration by parts to evaluate

$$\int x \sin(4x) dx.$$

[3 marks]

c) Given the curves $y = 2x^2 + 10$ and $y = 4x + 16$.

i) On the same axes sketch the curves and find the points of intersections between the curves.

[1.5 marks]

ii) Find the area of region enclosed by the two curves.

[2 marks]

Continued.....

Question 4 (15 marks)

a) Given the complex number $z = 2 - 2\sqrt{3}i$.

i) Graph z on the complex plane.

[1 mark]

ii) Then convert the complex number to polar form.

[3 marks]

iii) Use **De Moivre's Theorem** to find z^9 . Leave the answer in the form of

$$z^9 = a + bi \text{ where } a, b \in \mathbb{R}.$$

[2 marks]

b)

i) Use divergent test to decide whether the series $\sum_{n=1}^{\infty} \frac{n^3}{5n^3 + 3}$ converges.

Give your reason.

[2 marks]

ii) Use **root test** to determine whether the series $\sum_{n=1}^{\infty} \left(\frac{n^3}{5n^3 + 3} \right)^n$ converges.

[1 mark]

c)

i) State the values of p for which the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent.

[0.5 mark]

ii) Use $\sum_{n=1}^{\infty} \frac{1}{n^2}$ for comparison with the series $\sum_{n=1}^{\infty} \frac{n^2}{2 + n^4}$ to decide if the series

$$\sum_{n=1}^{\infty} \frac{n^2}{2 + n^4} \text{ is convergent or divergent. Show proper steps.}$$

[2 marks]

d) Find the Maclaurin polynomial of order 2 for $f(x) = e^{2x}$.

[3.5 marks]

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